International Journal of Emerging Trend in Engineering and Basic Sciences (IJEEBS)
ISSN (Online) 2349-6967
Volume 2 , Issue 1(Jan-Feb 2015), PP114-117

Design of Equiripple Low Pass FIR Filter

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Abstract:- The equiripple design produces the most efficient filters - that is, filters that just meet the specification with the least number of coefficients. The design of FIR filters using Windows methods leads to good performance filters. However, sometimes there is a need to design a FIR filter that not only performs well but it is optimal. Optimization is the ability to specify a maximum error on each band of interest. This error is expressed as the absolute difference between the ideal or desired frequency response and the actual or resulting frequency response. One of the techniques to design optimal FIR filters is to minimize a Chebyshev error criterion. The resulting filters are known as Equiripple FIR Filters.

Keywords:- Chebyshev, Equiripple FIR filter, Low Pass, Remez Exchange Algorithm, Remezord

I. INTRODUCTION

This paper explains how to design Equiriple Low pass FIR Filters. It covers some mathematical background necessary to understand how to evaluate and calculate the error function. The Remez Exchange Algorithm and its most common implementation by Parks, McClellan and Rabiner [1] are explained. Then, a practical guide to design Equiripple Low pass FIR filters is developed.

II. MATHEMATICAL BACKGROUND

Lets first begin by presenting a table of the four types of FIR filters [2][3].

Type	I	II	III	IV
Order	Even	Odd	Even	Odd
F(θ)	1	Cos(π/2)	Sin(θ)	Sin(θ/2)
M	N/2	(N-1)/2	(N-2)/2	(N-1)/2
θ_0	0	0	π/2	π/2

Table 1: Parameters of the four FIR filter types

In order to minimize the error we need to define an error function E(θ) and a weight function W(θ) which defines the relative importance of the error at any given frequency θ. Then, the error function can be described as follows:

$$E(\theta)=W(\theta)[A_d(\theta) - A(\theta)] \quad \text{(i)}$$

Where $A_d(\theta)$ is the desired amplitude response , and A(θ) is the actual amplitude response. A simple weight function W(θ) could be defined as follows:

$$W(\theta) = \begin{cases} 1 & \theta \in Passband \\ 0 & \theta \in Stopband \end{cases} \quad \text{(ii)}$$

And the resulting amplitude response A(θ) is defined by:

$$A(\theta)=F(\theta)G(\theta) \quad \text{(iii)}$$

and

$$G(\theta) = \sum_{k=0}^{M} b[k]\cos(k\theta) \quad \text{(iv)}$$

Where F(θ) and M are obtained from Table 1. The problem here is to obtain the coefficient b[k] that minimize the maximum absolute weighted error $|E(\theta)|$, that is, to obtain

$$\varepsilon = max|E(\theta)| \quad \text{(v)}$$

Where θ is the operating frequency range of the filter. The alternation theorem states that there exist at least K+2 frequencies θ_i, $\{0 \leq i \leq K+1\}$ where the maximum error, ε, occurs. That is,

$$|E(\theta_i)| = \varepsilon, \qquad 0 \leq i \leq K+1 \tag{vi}$$

and

$$E(\theta_{i+1}) = -E(\theta), \quad 0 \leq i \leq K \tag{vii}$$

The last equation shows that the sign changes K+1 times, resulting in an oscillation or ripple on the band of interest.

III. REMEZ EXCHANGE ALGOTITHM

The most common implementation of the Remez Exchange Algorithm is the version by Parks, McClellan and Rabiner [1][4]. Its objective is to obtain the coefficients b[k] that minimize ε. It uses the properties of the Alternation Theorem.

The first step is to find the order N of the desired filter. The following is empirical formulae proposed by Kaiser:

$$N = \frac{-20\log_{10}\left(\sqrt{\delta_p\delta_s}\right) - 13}{2.32|\theta_p - \theta_s|} \tag{viii}$$

Where

θ_p is the passband-edge digital frequency,

θ_s is the stopband-edge digital frequency,

δ_p is the passband allowed deviation,

δ_s is the stopband allowed deviation,

And

$$\delta_p = \frac{\left(10^{\frac{A_p}{20}} - 1\right)}{\left(10^{\frac{A_p}{20}} + 1\right)} \tag{ix}$$

$$\delta_s = 10^{-\frac{A_s}{20}} \tag{x}$$

where A_p and A_S are the attenuations on the passband and stopband respectively.

The following flowchart describes the steps required to implement the Remez Exchange Algorithm. In practice, the best way to design Equiripple FIR Filters is by using the functions **remezord** and **remez**[5] included in the Signal Processing Toolbox of the MATLAB.

Function **remezord** calculates the optimal filter order, N, and the optimal frequency points and relative weights. Function **remezord** has 4 input parameters:

- **f**, the vector of frequency-edges of the bands of interest (θ)
- **a**, the vector of band amplitudes (1 to indicate passband, 0 to indicate stopband)
- **dev**, the vector of allowed deviations on the bands (δ_p and δ_s in Equations (ix) and (x))
- **fs**, the sampling frequency

and returns the following output variables/vectors:

- **N**, order of the filter
- **f0**, vector of normalized frequency band edges (optimal points)
- **a0**, vector of frequency band amplitudes
- **w0**, vector of frequency band relative weights (optimal values for W(θ) in Equation (ii))

Function **remez** calculates the coefficients b[k] in Equation (iv). Its input parameters are exactly the output parameters of function **remezord**. Therefore, these 2 functions have to be used together, in sequence. Function **remez** has 4 input parameters:

- **N**, order of the filter
- **f0**, vector of normalized frequency band edges (optimal points)
- **a0**, vector of frequency band amplitudes
- **w0**, vector of frequency band relative weights (optimal values for W(θ) in Equation (ii))

and has one output parameter, **b**, the vector of filter coefficients b[k] in Equation (iv)

IV. ALGORITHM TO DESIGN EQUIRIPPLE LOW PASS FIR FILTER

1 User Input: Filter Type (LP)
2 User Input: Frequency Edges (vector f, depending on the filter type)
3 User Input: Sampling Frequency (fs)
4 User Input: Attenuation on the passband (A_p)
5 User Input: Attenuation on the passband (A_s)
6 Calculate δ_p and δ_s using Equations (ix) and (x) and populate vector **dev**.
7 If filter type is LP then a=[1 0]
8 Use the **remezord** function: [n,f0,a0,w] = remezord(f,a,dev,fs)
9 Use the **remez** function: b=remez(n,f0,a0,w)
10 Use the freqz function to obtain the h[k] coefficients
11 Plot the frequency response.

V. EXAMPLE OF EQUIRIPPLE FIR FILTER DESIGN

Filter Specifications:

Table: Low Pass Equiripple FIR Filter Specifications

Parameters	Values
Cutoff frequency	1000Hz
Stopband edge frequency	1200Hz
Sampling frequency	4000Hz
Passband attenuation	0.1dB
Stopband attenuation	40dB

VI. RESULTS & CONCLUSIONS

The frequency response plot in Figure 1 shows that the filter requirements were satisfied. However, the program had to be modified to increase the order of the filter. The first run of the program showed that the stopband attenuation requirement was not met (35dB as opposed to 40dB). It was found experimentally that the order of the filter has to be increased in 8 steps. The order of this filter is N=50. Figure 2 shows the plot of passband details.

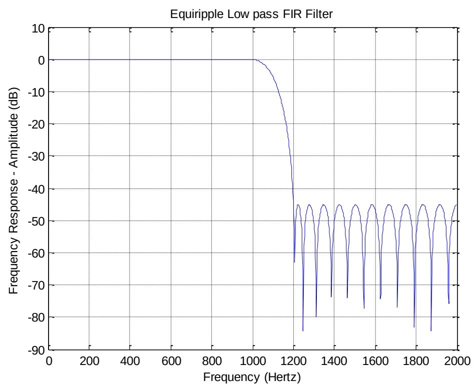

Figure.1: Frequency responses of Low pass Equiripple FIR Filter

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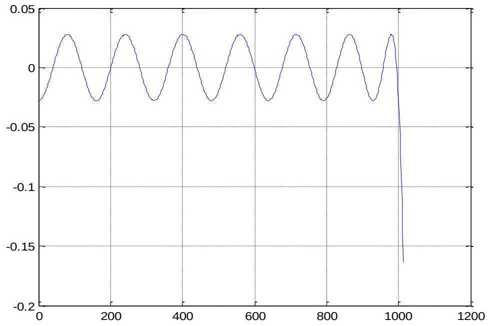

Figure.2: Equiripple Low Pass FIR Filter – Passband details

VII. CONCLUSIONS

The methodology to design Equiripple Low pass FIR Filters is simple and leads to good optimal FIR filters with respect to the Chebyshev norm. This technique allows the designer to explicitly control the band edges and relative ripple sizes on each band of interest. The order of the filter, N, obtained by using the function remezord does not yield the best results. Some experimentation is required to obtain the best value of N, the filter order. It was found that it is necessary to increase the order of the filter to meet the stopband attenuation requirement.

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